ON THE METHOD OF MOVING PLANES AND SLIDING DOMAINS

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OUTLINE

1. BACKGROUND

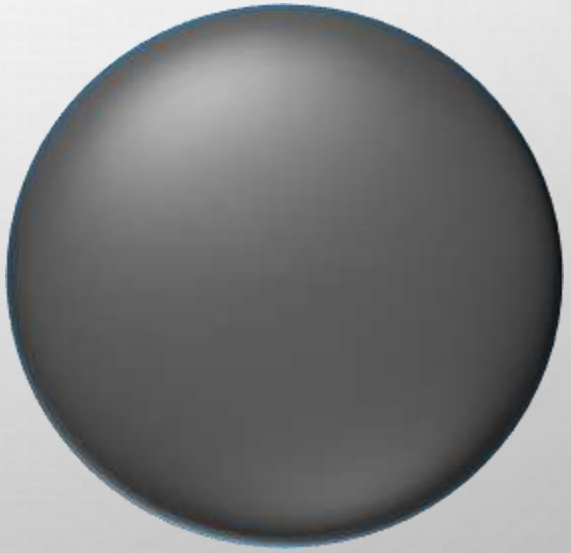
2. THE PROCEDURE OF MOVING PLANE METHOD

3. APPLICATIONS

BACKGROUND

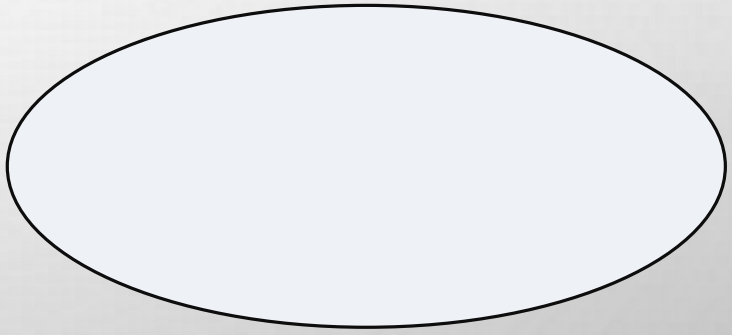
1. The method was introduced by A.D. Alexandov in the study of constant mean curvature of simple closed surface.

2. Any simple closed surface of class C^3 with constant mean curvature is a sphere.



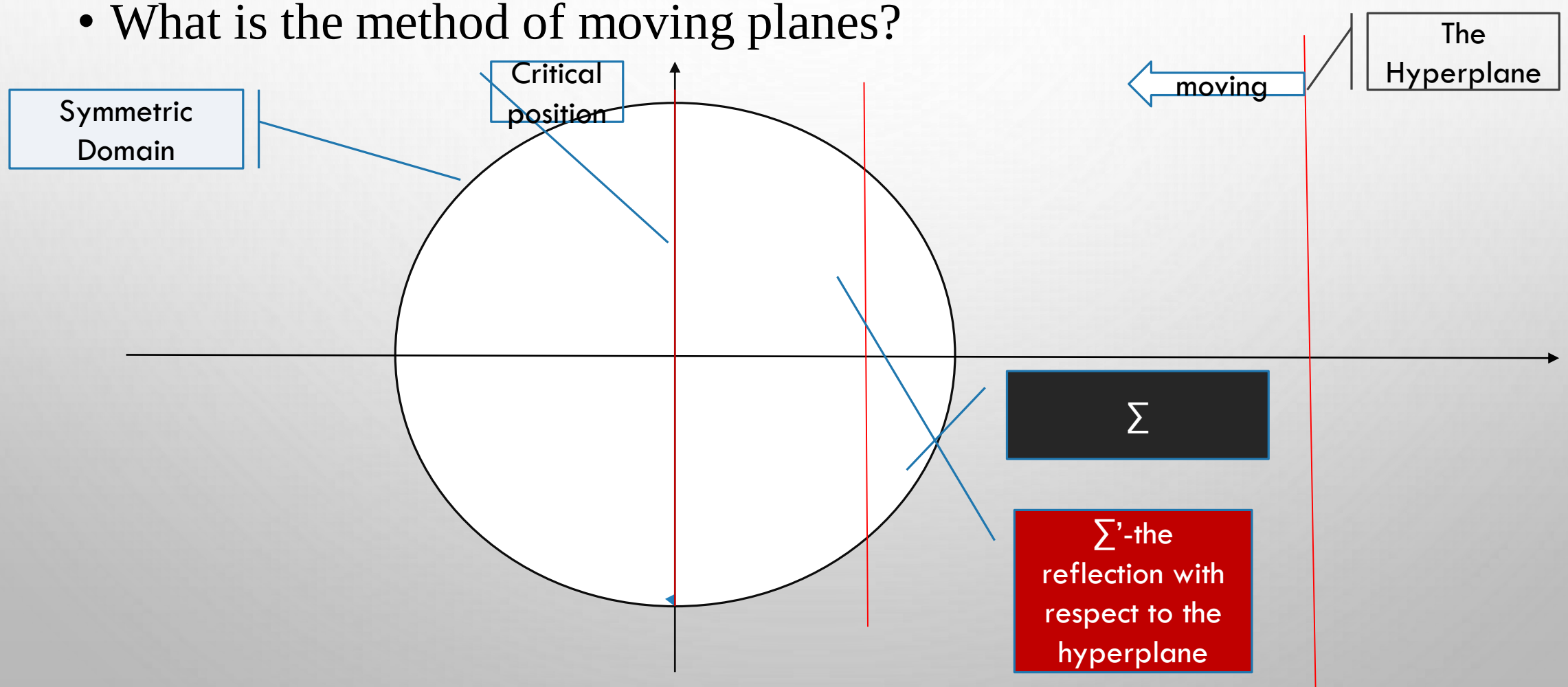
BACKGROUND

- Serrin first used the method to consider the solution of second order elliptic equations.
- Then, the method of moving planes was used by Gidas, Ni and Nirenberg in the proof of the symmetry or monotonicity of positive solutions.



THE PROCEDURE OF MOVING PLANE METHOD

- What is the method of moving planes?



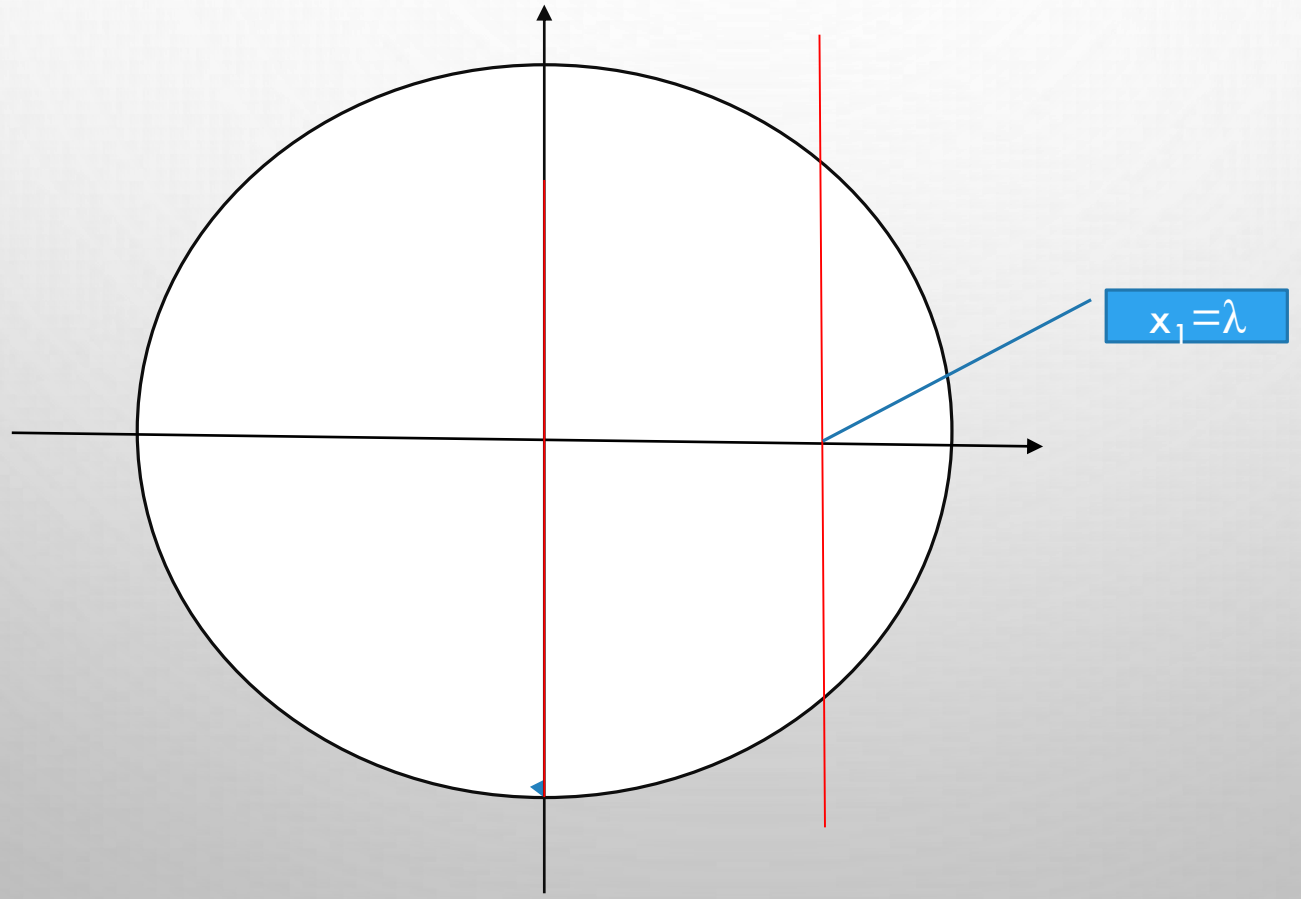
THE PROCEDURE OF MOVING PLANE METHOD

- The method compares two values. One is achieved at the point in $\sum$. The other is achieved at the reflection point in $\sum'$.
- The method has two steps:

  1. $w^{\lambda}>0$ holds when the moving plane is close to the boundary;
  2. Show that the moving plane stops at the critical position.

THE PROCEDURE OF MOVING PLANE METHOD

Define $w^{\lambda}=u(x^{\lambda})-u(x)$

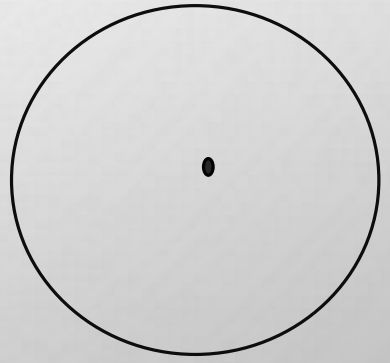


APPLICATIONS

- By the moving planes method, we can get some interesting results.
- For bounded domain:

If the domain is a ball $B_R(0)$, we have the uniqueness of the solution of:

$\Delta u + up = 0$ with $u = 0$ on $|x| = R$, $p > 1$



APPLICATIONS

- Another more interesting result is as follows:

For R^n, $n>2$, $1< p \leq n+2/n-2$:

$$\Delta u + u^p = 0$$

If $p<n+2/n-2$,

any nonnegative solutions $u=0$.

If $p=n+2/n-2$,

every positive solution is radially symmetric about some point.

THANKS!